Blood Supply Allocation: Trade-Off Between Equity and Efficiency

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Abstract

Blood supply allocation can involve a trade-off between equity and efficiency. While ABO-compatible pooling can promote equitable access across blood types by redistributing supply among all compatible blood types, it may reduce the total number of treated patients in a steady-state setting. This paper derives a domination criterion for ABO-compatible pooling and shows that in iterative pairwise pooling procedures, the final allocation does not result in efficiency loss if this criterion is met at each step. Simulations based on the plasma donation and demand model by Kominers et al. (2020) indicate that failing to satisfy this criterion leads to inefficiencies in 7.11% of cases, with the number of treated patients decreasing by an average of 0.60% to 1.03%. These findings highlight the need to integrate efficiency considerations into blood allocation mechanisms to prevent equity-driven pooling from inadvertently reducing the number of treated patients.

Keywords: Equity-Efficiency Trade-off; ABO-Compatibility; Blood Allocation; Market Design.

JEL Classification: D47; D63; C63; I10

1 Introduction

How should scarce blood resources be allocated to balance efficiency, maximizing lives saved, and equity, ensuring fair distribution? Blood is a scarce and perishable resource, and its allocation is constrained by biology, most notably by ABO blood type compatibility.[1] While some

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[1]Despite the regenerative nature of blood, shortages remain widespread. A modeling study by Roberts et al. (2019) estimated deficits in 119 of 195 countries (61.03%). The COVID-19 pandemic further exacerbated these

blood types are universal recipients, capable of receiving blood from any donor, others are limited to a subset of compatible blood types or even restricted to their own, creating asymmetries in supply and demand across blood types.[2] These imbalances are further shaped by the underlying variations in blood type distributions, which are influenced by factors such as geography, ethnicity and historical migration patterns. As a result, shortages disproportionately affect certain blood types and, in turn, specific sub-populations, widening disparities in access to life-saving blood resources.

Designing blood supply allocation mechanisms that balance efficiency and equity is crucial not only for improving health outcomes but also for mitigating the broader socio-economic consequences of unequal access to vital medical resources. While there is a substantial body of operations research literature on blood supply chain management, the focus there has been primarily on minimizing wastage (Pirabán et al., 2019). Although effective in routine settings, these models are less applicable in high-demand, supply-constrained situations, such as during viral pandemics, where shortages and allocation decisions pose the primary challenge. This issue became particularly evident during the COVID-19 pandemic, when convalescent plasma (CP) therapy, the practice of transfusing antibody-rich plasma from the blood of recovered individuals into infected patients to enhance immune response, emerged as an early treatment option. However, limited supply and biological compatibility constraints raised pressing questions about how to allocate it effectively.

Research on blood supply allocation has only recently been introduced in the field of market design, notably in the work of Kominers, Pathak, Sönmez, and Ünver (2020). They have taken a step toward addressing this gap by developing and analyzing a centralized mechanism for the collection and distribution of CP within the context of the COVID-19 pandemic. Of particular relevance to this paper, they argue that allowing ABO-compatible plasma allocation (i.e., where patients receive plasma from any ABO-compatible donor) can promote a more equitable distribution of supply across patients with different blood types and propose an optimal ABO-compatible pooling procedure. To the best of my knowledge, their study is the first and only to place an explicit emphasis on equity in blood supply allocation and to formalize an ABO-compatibility based blood pooling procedure.

This paper seeks to contribute to the nascent research on blood allocation in market design by building upon the work of Kominers et al. (2020) and further exploring the implications of allowing for ABO-compatible allocation. Specifically, in this paper I demonstrate that, in a steady state, pooling ABO-compatible blood types together could entail a potential trade-off between equity and efficiency objectives. Procedures aimed at equalizing the proportion of patients

shortages, with WHO reporting a 20–30% decline in global supply (Loua et al., 2021). Several countries, including Ireland (Horan, 2024), India (NACO, 2025), the United States (American Red Cross, 2024), the United Kingdom (Roxby, 2022), and Australia (Australian Red Cross Lifeblood, 2025), have faced record-low supply levels, prompting emergency donor appeals and cross-border imports.

[2]Details on blood type compatibility are presented in Section 2.

treated across different blood types could lead to a reduction in the total flow rate of treated patients. Additionally, I demonstrate that the final allocation in pairwise iterative blood pooling procedures is sensitive to the sequence in which pooling decisions are made due to ABO blood type compatibility constraints.

To address these challenges, I derive a criterion that ensures no efficiency loss in pairwise iterative pooling procedures and prove that as long as this criterion is met at each step, the final allocation weakly dominates ABO-identical treatment (i.e., where patients receive blood only from donors of the same blood type) in terms of total flow rate of treated patients whenever an alternative allocation is selected. Furthermore, I conduct a simulation study to quantify the magnitude of these theoretical findings. Using the CP supply and demand model of Kominers et al. (2020), I find that pooling procedures that violate this criterion result in efficiency losses in 7.11% of the generated samples, reducing the number of patients treated per unit time. On average, this reduction ranges from 0.60% to 1.03%, with maximum observed inefficiencies reaching up to 8.42% fewer patients treated.

The remainder of this paper is structured as follows. Section 2 provides essential background on blood transfusion and compatibility constraints. Section 3.1 presents the theoretical framework, while Section 4 discusses the motivation. Theoretical results are outlined in Section 5, followed by simulation results in Section 6. Section 7 reviews relevant literature, and Section 8 discusses policy implications and directions for future research.

2 Blood Transfusion and Compatibility

To understand the problem of interest, it is essential to establish a basic understanding of blood transfusion and the associated blood type compatibility constraints.

Blood consists of four main components: plasma, red blood cells, white blood cells, and platelets. Plasma, a yellowish liquid comprising about 55% of blood volume, contains electrolytes and proteins vital for nutrient distribution and immune response. Red blood cells, which make up approximately 44% of blood, are responsible for oxygen transport. White blood cells and platelets constitute the remaining 1% of blood, protecting against infections and aiding in clotting.

Blood transfusion is a procedure that replenishes lost or inadequate blood supply intravenously. In modern medical practice, whole blood is rarely transfused; instead, specific components are used depending on the patient's condition. Red blood cell transfusions are primarily used to treat anemia, often caused by cancer or other blood disorders. Platelet transfusions address thrombocytopenia (a low platelet count) and platelet function abnormalities. [3] Plasma transfusions are used to treat severe liver disease, bleeding disorders, serious burns, and, relevant to this paper, severe infections.

[3] White blood cell transfusions are rare due to the risk of virus transmission, which can lead to immune suppression and release toxic substances in the recipient (American Red Cross, 2022).

Beyond its routine medical use, plasma has also long played a vital role in combating emerging viral pandemics, serving as a first line of defense when vaccines and antiviral treatments are unavailable.[4] Convalescent plasma (CP), derived from recovered individuals and rich in virus-specific antibodies, is transfused into infected patients to enhance immune response. Most recently, CP was used during the COVID-19 pandemic, where a meta-analysis found it to be associated with a 30% reduction in hospitalization rates (Levine et al., 2023).[5]

Blood type compatibility is essential for determining whether a transfusion is feasible. Although there are currently 47 recognized blood group systems with 366 red cell antigens (International Society of Blood Transfusion, 2025), the ABO system is the most clinically significant.[6]

In the ABO system, blood types are defined by the presence or absence of A and B antigens on the surface of red blood cells. Blood type A has the A antigen, type B has the B antigen, type AB has both antigens, and type O has neither.

Table 1: ABO Blood Type Compatibility for Red Blood Cells and Plasma Transfusion

	Donor Blood Type	
Recipient Blood Type	Cell	Plasma
A	A, O	A, AB
AB	A, AB, B, O	AB
B	B, O	AB, B
O	O	A, AB, B, O

Each person's plasma contains antibodies against antigens not present on their red blood cells, which can trigger hemolysis (the destruction of red blood cells) if incompatible blood is transfused. Thus, ABO-compatibility is critical for red blood cell transfusion. Type O individuals, with antibodies against both A and B antigens, can only receive red blood cells from type

[4]While this practice dates back to the 1918 Spanish flu outbreak (Luke et al., 2006), CP therapy has also been used during the 2003 SARS-CoV-1 epidemic (Hui, 2013), the 2009–2010 H1N1 influenza pandemic (Beigel et al., 2019), the 2012–2013 MERS-CoV epidemic (Mo & Fisher, 2016), and the 2013–2016 Ebola epidemic (Van Griensven et al., 2016).

[5]Beyond its role at the onset of a viral outbreak, CP often serves as the most accessible and affordable treatment option for much of the world's population, particularly in regions with limited access to vaccines and antiviral treatments (Hill et al., 2021; Plata, 2022; Ramachandran et al., 2021). For instance, while CP can be more readily sourced from recovered donors, antiviral treatments for COVID-19 such as Paxlovid are priced at $1,390 per full course (Erman, 2023), and Molnupiravir costs £513 per course (Nuffield Department of Primary Care Health Sciences, 2024).

[6]The Rh system, another important blood group system in clinical practice, is based on the presence (Rh D+) or absence (Rh D−) of the Rh D antigen on red blood cells. Rh D+ individuals can receive blood from both Rh D+ and Rh D− donors, while Rh D− individuals can only receive Rh D− blood to avoid developing antibodies and hemolysis. Therefore, in addition to ABO-compatibility, Rh D− patients need Rh D-identical donors. While the Rh system is crucial in clinical practice, this paper focuses solely on ABO-compatibility for clarity, although the results can be generalized to include Rh compatibility.

O donors. Type AB individuals can receive red blood cells from any donor due to the absence of anti-A and anti-B antibodies. Type A individuals can receive blood from type A or O donors, while type B individuals can receive from type B or O donors.

Plasma transfusions follow different compatibility rules because plasma contains antibodies rather than antigens. Consequently, plasma from donors with anti-A or anti-B antibodies cannot donate to recipients who have those corresponding antigens. As a result, type AB plasma can be given to any blood type (as it lacks anti-A and anti-B antibodies), while type O plasma can only be given to type O patients. Table 1 summarizes ABO-compatibility for both red blood cell and plasma transfusions.

3 Model Preliminaries and Pooling Procedure

This section builds on the steady-state model of plasma transfusion and iterative pairwise pooling procedures under ABO-compatibility constraints developed by Kominers et al. (2020), adopting their notation and core framework. The first part summarizes their model, which describes how pooling ABO-compatible blood types can promote a more equitable allocation by equalizing the proportion of treated patients across the pooled blood types. The second part introduces a slightly modified version of their iterative pooling procedure, enabling analysis of how different pooling sequences affect final allocations in the following section. While the primary focus is on plasma allocation, the insights extend to other blood components with similar compatibility requirements.

3.1 Model Preliminaries

Let $\mathcal{B} = \{O, A, B, AB\}$ denote the set of all blood types. Given the critical role that ABO-compatibility plays in plasma transfusion, it is formally defined as follows:

Definition 1. *A blood type $Y \in \mathcal{B}$ is said to be **ABO-compatible** with another blood type $X \in \mathcal{B}$ if blood type Y plasma is compatible with patients with the blood type X.*

Assume a steady-state scenario where plasma transfusion patients with the blood types $X \in \mathcal{B}$ arrive at a rate of π_X per unit of time. Let σ_X represent the total plasma units available per unit time for patients with the blood type X, and δ_X denote the total plasma demand per unit time from patients with the blood type X, both of which are positive ($\sigma_X, \delta_X > 0$).[7]

Now, consider ABO-identical treatment, where patients receive transfusions only from donors with the same blood type. In contrast, ABO-compatible treatment allows plasma from other

[7]This restriction differs from the Kominers et al. (2020) model, which permits $\delta_X < 0$. In their paper, the underlying model determining demand and supply allows for scenarios where $\delta_X < 0$, indicating infinite supply. This means there is sufficient blood type X plasma to meet the needs of all patients, including those with compatible blood types.

ABO-compatible blood types to also be used. The supply-to-demand ratio for blood type X under ABO-identical treatment, denoted as s_X, is defined as:

$$s_X = \frac{\sigma_X}{\delta_X} \tag{1}$$

Here, $s_X > 1$ implies that there is an excess supply of blood type X plasma, and $s_X < 1$ implies that there is an excess demand.

The term $\min(s_X, 1)$ represents the proportion of blood types X patients who are able to receive treatment, as it measures the extent to which supply of type X plasma meets or exceeds demand. When $s_X > 1$, all patients with blood type X can be treated. The flow rate of treated blood type X patients is given by $\min(s_X, 1)\pi_X$. In cases where $s_X < 1$, only a fraction of the arriving patients can be treated.

Further, the average plasma demand per patient for blood type X is denoted as $\frac{\delta_X}{\pi_X}$. This ratio indicates the average number of plasma units required per blood type X patient.

The supply-to-demand ratio (s_X) can vary across the different blood types. Such variation may arise naturally from the underlying population's blood type distribution, where some blood types are rarer than others (Give Blood, 2022). It might also, as noted by Kominers et al. (2020), be exacerbated by circumstantial factors. For example, during the COVID-19 pandemic, variations in outbreak locations and differences in people's ability to practice social distancing led to higher infection rates in certain regions and among specific ethnic groups.

Such shortages can be partially alleviated by adopting ABO-compatible treatment, a practice that is already implemented in healthcare systems. For example, consider a market for convalescent plasma where the supply-to-demand ratio for type O plasma is lower than that for type A, i.e., $s_O < s_A$, and there is a shortage of blood type O plasma, i.e., $s_O < 1$.[8] Blood type A is ABO-compatible with blood type O, i.e., type O patients can receive blood type A plasma (see Table 1). By directing some blood type A plasma to blood type O patients, the supply-to-demand ratio for blood type O patients can be increased.

Plasma Transfer and Supply-to-Demand Ratio Equalization

Let the number of units of blood type A plasma transferred to blood type O patients be denoted by $\sigma_{A\to O}$. After this transfer, the new supply-to-demand ratios for the two blood types become:

$$s_O = \frac{\sigma_O + \sigma_{A\to O}}{\delta_O} \leq \frac{\sigma_A - \sigma_{A\to O}}{\delta_A} = s_A \tag{2}$$

The number of units transferred from type A to type O can be increased until the supply-to-demand ratio for both blood types are equal. This process either results in both blood types

[8]For our purpose, it does not matter if s_A is also less than 1.

have an excess supply (i.e., $s_O, s_A > 1$, meaning all patients are treated) or both having equal supply-to-demand ratios that remain below 1 (i.e., $s_O = s_A < 1$, meaning both blood types face shortages).

The amount of plasma transfer required to equalize the supply-to-demand ratio for both blood types is given by:

$$\sigma_{A\to O} = \frac{\sigma_A\delta_O - \sigma_O\delta_A}{\delta_O + \delta_A} \tag{3}$$

After this transfer, the resulting pooled supply-to-demand ratio for both blood type is:

$$s_{\{O,A\}} \coloneqq \frac{\sigma_O + \sigma_A}{\delta_O + \delta_O} = s_O = s_A \tag{4}$$

Additionally, the pooled ratio satisfies the following property:

$$s_O = \frac{\sigma_O}{\delta_O} < s_{\{O,A\}} = \frac{\sigma_O + \sigma_A}{\delta_O + \delta_A} < \frac{\sigma_A}{\delta_A} = s_A \tag{5}$$

In other words, the pooled supply-to-demand ratio falls between the original ratios of the two blood types. This transfer process reduces the disparity in supply, redistributing the shortage or surplus more equitably between the two blood types.

This reallocation process can be extended to include more than two blood types. For instance, if the supply-to-demand ratio of blood type B is greater than that of type O, then type B plasma can also be transferred to type O patients. Simultaneously, some of the type A plasma can be reallocated back to type A patients to balance their respective supply-to-demand ratios. The pooled supply-to-demand ratio for the blood types would then be:

$$s_{\{O,A\}} < s_{\{O,A,B\}} = \frac{\sigma_O + \sigma_A + \sigma_B}{\delta_O + \delta_A + \delta_B} < s_B \tag{6}$$

Equality in the supply-to-demand ratios can always be achieved due to the continuous nature of the plasma transfer process. As plasma is incrementally transferred between blood types, the supply-to-demand ratios adjust smoothly. By monotonicity and the Intermediate Value Theorem, there must be a point where the two ratios become equal. The amount of plasma transfer required to achieve this balance, $\sigma_{A\to O}$, is well-defined for $\delta_O, \delta_A > 0$, ensuring that equalization can occur regardless of whether the initial conditions indicate excess supply or shortages. This process naturally extends to multiple blood types, guaranteeing that equal ratios can be always be attained through appropriate reallocation.

Henceforth, the term *pooling* will refer to the process of equalizing the supply-to-demand

ratios among blood types and the set of blood types for which the supply-to-demand ratios have been made equal will be referred to as the *set of pooled blood types*.

3.2 Iterative Pairwise Pooling Procedure

To formalize the intuitive process described above, an iterative pairwise pooling procedure is introduced. It involves constructing a sequence of subsets of pooled blood types. In each iteration, the blood type or pooled subset with the lowest supply-to-demand ratio, s_X, is selected, and a decision is made on whether to pool it with another compatible blood type or pooled subset of blood types, chosen according to a preselected rule, $\mathbf{R}$.[9]

To formally define the procedure, the concept of compatibility set is introduced, identifying feasible pooling options at each iteration. For any $\mathcal{X} \subseteq \mathcal{B}$ and any collection of subsets $\mathbf{Y} \subseteq 2^{\mathcal{B}}$, the **compatibility set** is defined as:

$$\mathcal{C}(\mathcal{X}, \mathbf{Y}) := \{\mathcal{Y} \in \mathbf{Y} : \exists\, X \in \mathcal{X} \text{ and } Y \in \mathcal{Y} \text{ such that } \mathbf{Y} \text{ is ABO-compatible with } X\}.$$

The considered procedure entails an iterative construction of a sequence $\{\mathbf{B}^t\}$, where each $\mathbf{B}^t \subseteq 2^{\mathcal{B}}$ represents a collection of subsets of blood types. Each set $\mathcal{X} \in \{\mathbf{B}^t\}$ is considered a pooled set, in which the supply-to-demand ratio of all blood types in $\mathcal{X}$ can be made equal. This is achieved by treating each blood type in $\mathcal{X}$ either with its own type or with some other compatible blood type $Y \in \mathcal{X}$. The pooled supply-to-demand ratio, $s_{\mathcal{X}}$, is defined as:

$$s_{\mathcal{X}} = \frac{\sum_{X \in \mathcal{X}} \sigma_X}{\sum_{X \in \mathcal{X}} \delta_X}.$$

The steps of the proposed iterative pooling procedure is then executed as follows:

Step 0: Let $\mathbf{B}^0 = \mathcal{B}$, the set of all blood types. Continue to next step.[10]

$\vdots$

Step t ≥ 1**:** Suppose $\mathbf{B}^{t-1}$ is the collection of pooled blood sets determined in the previous step. Let $\mathcal{X} \in \mathbf{B}^{t-1}$ be the pooled set with lowest service rate with $s_{\mathcal{X}} < 1$ (i.e., there is a shortage of blood for the blood types in the set $\mathcal{X}$). Then:

[9] This procedure is a slight modification of the one originally proposed by Kominers et al. (2020). In the original proposal, the blood type or pooled subset with the lowest supply-to-demand ratio is selected, and a decision is made on whether to pool it with another compatible blood type or pooled subset of types with the highest supply-to-demand ratio. In this version, the second blood type or subset is selected using a preselected arbitrary function.

[10] Kominers et al. (2020) begin their procedure by setting $\mathbf{B}^0 \subseteq \mathcal{B}$ to exclude ABO-compatible blood types that have access to an infinite supply. However, the possibility of an infinite supply pertains to the specifics of their model and is not relevant to this context.

- If $\mathbf{C}(\mathcal{X}, \mathbf{B}^{t-1}) \supsetneq \{\mathcal{X}\}$, an element $\mathcal{Y}$ is selected from the compatibility set according to the preselected rule $\mathbf{R}$. Then $\mathcal{X}$ and $\mathcal{Y}$ are pooled together and replaced with their union $\mathcal{S} = \mathcal{X} \cup \mathcal{Y}$, so that:

  $\mathbf{B}^t := (\mathbf{B}^{t-1} \backslash \{\mathcal{X}, \mathcal{Y}\}) \cup \{\mathcal{S}\}$.

  Set the new supply to demand ratio for $\mathcal{S}$ as:

  $s_{\mathcal{S}} := \frac{\sum_{X \in \mathcal{S}} \sigma_X}{\sum_{X \in \mathcal{S}} \delta_X}$

- If $\mathbf{C}(\mathcal{X}, \mathbf{B}^{t-1}) = \{\mathcal{X}\}$, then $\mathcal{X}$ cannot be pooled with any other set, the supply-to-demand ratio for each blood type $X \in \mathcal{X}$ is set as $s_{\mathcal{X}}$ and:

  $\mathbf{B}^t := \mathbf{B}^{t-1} \backslash \{\mathcal{X}\}$

If $\mathbf{B}^t = \emptyset$, then any final supply-to-demand ratio greater than 1 is set to 1 and the process terminates. Otherwise, continue to the next iteration, step $t + 1$.

The allocation resulting from this pooling procedure may be desirable from an equity standpoint, as it reduces disparities in plasma access among patients with different blood types, ensuring that the same proportion of patients is treated across all pooled blood types. However, while pooling ABO-compatible blood types promotes equity, it may also lower the total flow rate of treated patients, raising a potential efficiency-equity trade-off. Moreover, the order of pooling decisions influences the final allocation and, consequently, the total flow rate of treated patients. Both factors are discussed in further detail in the following section.

4 Motivation

The objective that a policymaker seeks to maximize in an allocation is often shaped by underlying normative principles. A utilitarian approach, as popularized by Bentham (1781), advocates for actions that produce the greatest good for the greatest number. In this context, one interpretation could be that the allocation should prioritize treating the largest possible number of patients, regardless of their blood type. In this paper, this objective is characterized using the notion of domination, as defined below.

Definition 2. *For a given subset of blood types $\mathcal{X} \subseteq \mathcal{B}$, a pooled allocation is said to be **dominant** if it results in a higher total flow rate of treated patients than ABO-identical treatment i.e.,*

$$\min\left(\frac{\sigma_{\mathcal{X}}}{\delta_{\mathcal{X}}}, 1\right) \sum_{X \in \mathcal{X}} \pi_X > \sum_{X \in \mathcal{X}} \min\left(\frac{\sigma_X}{\delta_X}, 1\right) \pi_X.$$

Here, $\sigma_{\mathcal{X}} = \sum_{X \in \mathcal{X}} \sigma_X$ and $\delta_{\mathcal{X}} = \sum_{X \in \mathcal{X}} \delta_X$, where σ_X, δ_X, and π_X denote the supply, demand, and arrival rates, respectively, as defined earlier.

It should be noted, however, that while related, the concept of domination used in this paper does not strictly require maximizing the total flow rate of treated patients. Rather, it establishes a minimum criterion where pooling should at least maintain a total flow rate of treated patients that is no lower than under ABO-identical treatment, ensuring no efficiency loss.

In contrast, a Rawlsian approach, centered on principles of justice and fairness, prioritizes improving outcomes for the worst-off individuals (Rawls, 1971). In the context of blood supply allocation, the focus may be on ensuring equitable access to plasma resources across blood types, prioritizing those with the least access, aligning with the procedure outlined in the preceding section. This notion is formalized as follows:

Definition 3. *For a given subset of blood types $\mathcal{X} \subseteq \mathcal{B}$, an allocation is said to be **equitable** if the same proportion of patients are treated across all blood types within the subset. This condition holds if either :*

1. *all blood types in the subset have an excess supply (i.e., $\sigma_X \geq \delta_X \; \forall \; X \in \mathcal{X}$), or*
2. *the supply-to-demand ratio is equal across all blood types within the subset (i.e. $\frac{\sigma_X}{\delta_X} = \frac{\sigma_Y}{\delta_Y}$ $\forall \; X, Y \in \mathcal{X}$).*

Here, σ, δ, and π denote the supply, demand and arrival rates, respectively, as defined earlier.

The following subsections examine two key issues related to these normative considerations, which motivate the theoretical results in this paper. First, in a steady state, pooling ABO-compatible blood types may create a trade-off between sustaining the total flow rate of treated patients and minimizing disparities in resource access across blood types—an efficiency-equity trade-off commonly observed in resource allocation. Second, the final allocation in pairwise iterative pooling procedures depends on the sequence of pooling decisions, which in turn influences the total number of patients treated.

4.1 The Efficiency-Equity Trade-off

This section provides a numerical illustration of the potential trade-off that may arise between sustaining the total number of treated patients and achieving equitability. Consider a scenario where the supply-to-demand ratio for blood type O patients is lower than that for blood type A patients. Suppose, in the steady state, there are 50 units of type O plasma (σ_O) available, 100 units of type A plasma (σ_A) available per unit time, and that the total demand from type O and type A patients is 150 units (δ_O) and 50 units (δ_A), respectively. This results in supply-to-demand ratios of $s_O = \frac{50}{150} \approx 0.33$ for type O and $s_A = \frac{100}{50} = 2$ for type A. Additionally, assume that the arrival flow rate of patients ($\pi_O + \pi_A$) is 50.

Pooling these blood types would yield a pooled supply-to-demand ratio of:

$$s_{\{O,A\}} = \frac{\sigma_O + \sigma_A}{\delta_O + \delta_A} = \frac{50 + 100}{150 + 50} = 0.75.$$

Pooling allows excess type A plasma to be redistributed to type O patients. However, the effect of this reallocation on the total flow rate of treated patients depends on the arrival rates of patients by blood type, as illustrated by the examples below.[11]

Table 2: Impact of Pooling on the Flow Rate of Treated Patients (Patients per Unit Time)

	Arrival Flow Rate		Treated Patient Flow Rate	
	π_O	π_A	Without Pooling	With Pooling
Example 1	25	25	33	37
Example 2	40	10	23	37
Example 3	10	40	43	37

Example 1 (Equal Arrival Flow Rates)**.** *Suppose $\pi_O = \pi_A = 25$. Without pooling, all blood type A patients are treated as there is a surplus of blood type A plasma, while only 33% of those with blood type O are treated. This results in a total flow rate of 33 treated patients (25 type A and 8 type 0). By pooling the blood types, the total flow rate increases to 37, as 75% of all patients are treated. This is achieved by reallocating some of the excess type A plasma to type O patients, thereby resulting in a dominant allocation.*

Example 2 (Higher Arrival Flow Rate for Type O)**.** *Suppose $\pi_O = 40$ and $\pi_A = 10$. Without pooling, there is a total flow rate of 23 treated patients (all 10 type A and 13 type O) due to the shortage of type O plasma. By pooling the blood types, the total flow rate increases to 37, marking a dominant allocation in this case as well.*

Example 3 (Higher Arrival Flow Rate for Type A)**.** *Suppose $\pi_O = 10$ and $\pi_A = 40$. Without pooling, there is a total flow rate of 43 treated patients (all 40 type A and 3 type O). However, pooling results in efficiency loss, reducing the total flow rate to 37 patients per unit time.*

In the three examples above, summarized in Table 2, pooling increased the total flow rate of treated patients in the first two cases but decreased it in the third. While pooling leads to an equitable allocation, ensuring that the same proportion of patients are treated across all blood types, it can also reduce the total flow rate of treated patients under certain conditions. These examples illustrate the potential trade-off between sustaining the total flow rate of treated patients and achieving equity when pooling ABO-compatible blood types in a steady-state setting.

[11] Although, the term *pooling* is used to describe this process, the actual transfer here is unidirectional. Specifically, 62.5 units of type A plasma would be reallocated to type O patients in order to balance the supply across the two groups.

4.2 Importance of Pooling Order

This section illustrates how the final allocation in pairwise iterative pooling procedure is sensitive to the order in which decisions are made, i.e. it depends crucially on the rule $\mathbf{R}$, due to ABO-compatibility constraints. To demonstrate this, consider an initial scenario where patients of different blood types have the following supply-to-demand ratios for plasma:

$$s_O = \frac{10}{40} = 0.25 < s_A = \frac{20}{40} = 0.50 < s_{AB} = \frac{32}{40} = 0.80 < s_B = \frac{100}{100} = 1$$

Two different rules ($\mathbf{R}$) for selecting the element from the compatibility set will now be considered. The first rule selects the element with the lowest supply-to-demand ratio, and the second selects the one with the highest supply-to-demand ratio.

CASE A: Lowest Supply-to-Demand Ratio Rule

$$\mathbf{R} = \{\mathcal{Y} \in \mathcal{C}(\mathcal{X}, \mathbf{Y}) \mid s_{\mathcal{Y}} = \min\{s_{\mathcal{Z}} \mid \mathcal{Z} \in \mathcal{C}(\mathcal{X}, \mathbf{Y})\}\}$$

The rule $\mathbf{R}$ selects the element $\mathcal{Y}$ from the compatibility set $\mathcal{C}(\mathcal{X}, \mathbf{Y})$ such that $s_{\mathcal{Y}}$ is the minimum supply-to-demand ratio among all elements in the compatibility set. The pooling procedure would then proceed as follows:

Step 0: $\mathbf{B^0} = \{\{A\}, \{AB\}, \{B\}, \{O\}\}$.
The supply to demand ratio are $s_O = 0.25, s_A = 0.50, s_{AB} = 0.80$ and $s_B = 1$.

Step 1: $\{O\}$ has the lowest supply-to-demand ratio.
$\mathcal{C}(\{O\}, \mathbf{B^0}) = \{\{A\}, \{AB\}, \{B\}, \{O\}\}$.
$\{A\} \in \mathcal{C}(\{O\}, \mathbf{B^0})\backslash\{\{O\}\}$ has the lowest supply-to-demand ratio.
$\{O\}$ and $\{A\}$ are pooled together as $\{O, A\}$.
$\mathbf{B^1} = \mathbf{B^0}\backslash\{\{O\}, \{A\}\} \cup \{O, A\} = \{\{AB\}, \{B\}, \{O, A\}\}$.
The new supply to demand ratio for $\{O, A\}$ is:

$$s_{\{O,A\}} = \frac{10 + 20}{40 + 40} = \frac{30}{80} = 0.38$$

Step 2: $\{O, A\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{O, A\}, \mathbf{B^1}) = \{\{AB\}, \{B\}, \{O, A\}\}$.
$\{AB\} \in \mathcal{C}(\{O, A\}, \mathbf{B^1})\backslash\{\{O, A\}\}$ has the lowest supply-to-demand ratio.
$\{O, A\}$ and $\{AB\}$ are pooled together as $\{O, A, AB\}$.
$\mathbf{B^2} = \mathbf{B^1}\backslash\{\{O, A\}, \{AB\}\} \cup \{O, A, AB\} = \{\{B\}, \{O, A, AB\}\}$.

The new supply to demand ratio for $\{O, A, AB\}$ is:

$$s_{\{O,A,AB\}} = \frac{30+32}{80+40} = \frac{62}{120} = 0.52$$

Step 3: $\{O, A, AB\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{O, A, AB\}, \mathbf{B^2}) = \{\{B\}, \{O, A, AB\}\}$.
$\{B\} \in \mathcal{C}(\{O, A, AB\}, \mathbf{B^2}) \backslash \{\{O, A, AB\}\}$ has the lowest supply-to-demand ratio.
$\{O, A, AB\}$ and $\{B\}$ are pooled together as $\{O, A, AB, B\}$.
$\mathbf{B^3} = \mathbf{B^3} \backslash \{\{O, A, AB\}, \{B\}\} \cup \{O, A, AB, B\} = \{\{O, A, AB, B\}\}$.
The new supply to demand ratio for $\{O, A, AB, B\}$ is:

$$s_{\{O,A,AB\}} = \frac{62+100}{120+100} = \frac{162}{220} = 0.74$$

Step 4: $\{O, A, AB, B\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{O, A, AB, B\}, \mathbf{B^3}) = \{\{O, A, AB, B\}\}$ so it is not pooled with any other set.
$\mathbf{B^4} = \mathbf{B^3} \backslash \{O, A, AB, B\} = \emptyset$.

The procedure ends and the pooled set is $\{O, A, AB, B\}$ with $s_{\{O,A,AB,B\}} = 0.74$.

CASE B: Highest Supply-to-Demand Ratio Rule

$$\mathbf{R} = \{\mathcal{Y} \in \mathcal{C}(\mathcal{X}, \mathbf{Y}) \mid s_{\mathcal{Y}} = \max\{s_{\mathcal{Z}} \mid \mathcal{Z} \in \mathcal{C}(\mathcal{X}, \mathbf{Y})\}\}$$

The rule $\mathbf{R}$ selects the element $\mathcal{Y}$ from the compatibility set $\mathcal{C}(\mathcal{X}, \mathbf{Y})$ such that $s_{\mathcal{Y}}$ is the maximum supply-to-demand ratio among all elements in the compatibility set.

Detailed procedural steps are provided in Annex B. Following this rule, the procedure results in two seperate pooled sets: $\{A, AB\}$ with a supply-to-demand ratio of $s_{\{A,AB\}} = 0.65$, and $\{O, B\}$ with a supply-to-demand ratio $s_{\{O,B\}} = 0.79$.

By contrast, in Case A, the procedure concluded with a single pooled set $\{O, A, AB, B\}$ and a supply-to-demand ratio of $s_{\{O,A,AB,B\}} = 0.74$. These examples illustrate how the final allocation in the iterative pooling procedure is sensitive to the rule $\mathbf{R}$ used to select the element from the compatibility set, or in other words, it is sensitive to the order in which the blood groups are pooled.

5 Theoretical Results

The preceding sections illustrated how pooling can affect the total flow rate of treated patients, and demonstrated that the final outcomes of ABO-compatible iterative pairwise pooling procedures are sensitive to the order in which the pooling decisions are made. This section presents

theoretical results that seek to address these issues. First, it identifies the conditions under which a pooled allocation remains dominant when pooling two blood types (Propositions 1–2) or subsets of pooled blood types (Propositions 3–4). Then, it establishes how this condition can be incorporated into any pairwise iterative pooling procedure to ensure dominant allocations, regardless of pooling order (Proposition 5).

Dominance in Pairwise Pooling of Blood Types

To formally analyze the outcomes of pairwise pooling, it is essential to establish the conditions under which reallocating supply from one blood type to another is both feasible and reasonable for alleviating shortages. Pooling may be relevant when a shortage exists, the blood types are ABO-compatible, and the blood type receiving the reallocated supply has a lower supply-to-demand ratio, resulting in a smaller proportion of its patients receiving treatment under ABO-identical allocation. These criteria align with equity considerations, as discussed in the preceding sections.

Formally, let X and $Y \in \mathcal{B}$ be two blood types such that:

1. Y is ABO-compatible with X, i.e., $Y \in \mathcal{C}(\{X\}, \mathbf{Y})$,
2. There is a shortage of blood type X, i.e., $\frac{\sigma_X}{\delta_X} < 1$, and
3. The supply-to-demand ratio of blood type Y is greater than that of blood type X, i.e. $\frac{\sigma_X}{\delta_X} < \frac{\sigma_Y}{\delta_Y}$.

When pooling X and Y, the pooled supply-to-demand ratio depends on the relative magnitudes of $\sigma_X, \sigma_Y, \delta_X$ and δ_Y. There are three potential outcomes:

Case I: $\frac{\sigma_X}{\delta_X} < 1 \leq \frac{\sigma_X+\sigma_Y}{\delta_X+\delta_Y} < \frac{\sigma_Y}{\delta_Y}$, i.e., when there is a sufficient supply of Y to fully cover both Y's demand and the shortage in X.

Case II: $\frac{\sigma_X}{\delta_X} < \frac{\sigma_X+\sigma_Y}{\delta_X+\delta_Y} < \frac{\sigma_Y}{\delta_Y} < 1$, i.e., when Y's supply-to-demand ratio is greater than X's, but the supply of both blood types remains insufficient to meet their respective demands.

Case III: $\frac{\sigma_X}{\delta_X} < \frac{\sigma_X+\sigma_Y}{\delta_X+\delta_Y} < 1 \leq \frac{\sigma_Y}{\delta_Y}$, i.e., when Y has sufficient supply to cover its own demand, but not enough to fully eliminate shortage in X.

In the first case, pooling trivially results in an efficient allocation. However, in the latter two cases the pooled supply-to-demand ratio remain below 1, meaning the shortages persists to some extent. These scenarios require further analysis to determine when pooling can still improve allocation outcomes. The following proposition establishes the necessary and sufficient condition for dominance in Case II.

Proposition 1. *When the supply-to-demand ratio of Y is greater than that of X (i.e., $\frac{\sigma_X}{\delta_X} < \frac{\sigma_Y}{\delta_Y}$), but both blood types are experiencing shortages (i.e. $\frac{\sigma_X}{\delta_X}, \frac{\sigma_Y}{\delta_Y} < 1$), pooling is dominant if and*

only if the average plasma demand per patient for blood type Y is greater than that for blood type X, i.e., $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$.

Proof. Pooling is dominant if the total flow rate of treated patients after pooling exceeds that under ABO-identical treatment. This condition can be expressed as:

$$\min\Big(\frac{\sigma_X}{\delta_X}, 1\Big)\pi_X + \min\Big(\frac{\sigma_Y}{\delta_Y}, 1\Big)\pi_Y < \min\Big(\frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}, 1\Big)(\pi_X + \pi_Y).$$

Since both X and Y are experiencing shortages ($\frac{\sigma_X}{\delta_X}, \frac{\sigma_Y}{\delta_Y} < 1$), the minimum functions simplify to:

$$\frac{\sigma_X}{\delta_X}\pi_X + \frac{\sigma_Y}{\delta_Y}\pi_Y < \frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}(\pi_X + \pi_Y).$$

Rearranging and grouping terms gives:

$$\frac{\sigma_X}{\delta_X}(\pi_X(\delta_X + \delta_Y) - \delta_X(\pi_X + \pi_Y)) < \frac{\sigma_Y}{\delta_Y}(\delta_Y(\pi_X + \pi_Y) - \pi_Y(\delta_X + \delta_Y)).$$

Simplifying and rearranging yields:

$$\Big(\frac{\sigma_X}{\delta_X} - \frac{\sigma_Y}{\delta_Y}\Big)(\delta_Y\pi_X - \pi_Y\delta_X) < 0.$$

Since $\frac{\sigma_X}{\delta_X} < \frac{\sigma_Y}{\delta_Y}$ (by assumption), the inequality above holds if and only if:

$$\delta_Y\pi_X > \pi_Y\delta_X \quad \text{or equivalently,} \quad \frac{\delta_Y}{\pi_Y} > \frac{\delta_X}{\pi_X}.$$

Thus, pooling is dominant if and only if the average plasma demand per patient for blood type Y is greater than that for blood type X. □

Therefore, even when both blood types are experiencing shortages, pooling can lead to an allocation that is both equitable and dominant, provided the condition $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$ is satisfied. Furthermore, this condition also serves as a sufficient criterion for dominance in Case III, where Y has an excess supply but not enough to fully eliminate the shortage in $X \cup Y$.

Proposition 2. *When there is an excess of Y (i.e., $\frac{\sigma_Y}{\delta_Y} > 1$), but not enough to fully eliminate the shortage in $X \cup Y$ (i.e. $\frac{\sigma_X+\sigma_Y}{\delta_X+\delta_Y} < 1$), pooling results in a dominant allocation if the average plasma demand per patient with blood type X is less than that for blood type Y, i.e., $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$.*

Proof. Pooling is dominant if the total flow rate of treated patients after pooling exceeds that under ABO-identical treatment, which is expressed as:

$$\min\left(\frac{\sigma_X}{\delta_X}, 1\right)\pi_X + \min\left(\frac{\sigma_Y}{\delta_Y}, 1\right)\pi_Y < \min\left(\frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}, 1\right)(\pi_X + \pi_Y).$$

Since Y has an excess ($\frac{\sigma_Y}{\delta_Y} > 1$) and X has an shortage ($\frac{\sigma_X}{\delta_X} < 1$), the min functions simplify to:

$$\frac{\sigma_X}{\delta_X}\pi_X + \pi_Y < \frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}(\pi_X + \pi_Y).$$

Since $\pi_Y < \frac{\sigma_Y}{\delta_Y}\pi_Y$ as $\frac{\sigma_Y}{\delta_Y} > 1$, we have:

$$\frac{\sigma_X}{\delta_X}\pi_X + \pi_Y < \frac{\sigma_X}{\delta_X}\pi_X + \frac{\sigma_Y}{\delta_Y}\pi_Y.$$

Further, following the same steps as in Proposition 1, it can be shown that:

$$\frac{\sigma_X}{\delta_X}\pi_X + \frac{\sigma_Y}{\delta_Y}\pi_Y < \frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}(\pi_X + \pi_Y).$$

can be rearranged and simplified to yield:

$$\Big(\frac{\sigma_X}{\delta_X} - \frac{\sigma_Y}{\delta_Y}\Big)(\delta_Y\pi_X - \pi_Y\delta_X) < 0.$$

As before, this inequality holds if and only if:

$$\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}.$$

Thus, combining the results, if $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$, then:

$$\frac{\sigma_X}{\delta_X}\pi_X + \pi_Y < \frac{\sigma_X}{\delta_X}\pi_X + \frac{\sigma_Y}{\delta_Y}\pi_Y < \frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}(\pi_X + \pi_Y).$$

Therefore, if $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$ pooling results in a dominant allocation. □

However, in Case III, while $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$ is sufficient for dominance, it is not necessary. The following numerical example demonstrates that pooling can still increase the total flow rate of treated patients even when this condition is violated.

Example (Case III: When the Condition Is Sufficient but Not Necessary)**.** *Consider the following scenario where Y has an excess supply ($\frac{\sigma_Y}{\delta_Y} > 1$), but this surplus is insufficient to fully eliminate the shortage in X ($\frac{\sigma_X+\sigma_Y}{\delta_X+\delta_Y} < 1$).*

Suppose blood type X has 80 units of plasma available ($\sigma_X = 80$), with a demand of 160 units ($\delta_X = 160$) and an arrival rate of 30 patients per unit time ($\pi_X = 30$). For blood type Y, assume 90 units of plasma available ($\sigma_Y = 90$), with a demand of 70 units ($\delta_Y = 70$) and an arrival rate of 20 patients per unit time ($\pi_Y = 20$).

The supply-to-demand ratios under ABO-identical allocation are:

$$\frac{\sigma_X}{\delta_X} = \frac{80}{160} = 0.5, \quad \textit{and} \quad \frac{\sigma_Y}{\delta_Y} = \frac{90}{70} \approx 1.29.$$

Pooling these two blood types results in a pooled supply-to-demand ratio of:

$$\frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y} = \frac{170}{230} \approx 0.74.$$

Thus, the total flow rate of treated patients under ABO-identical treatment is:

$$\min\left(\frac{\sigma_X}{\delta_X}, 1\right)\pi_X + \min\left(\frac{\sigma_Y}{\delta_Y}, 1\right)\pi_Y = (0.5 \times 30) + 20 = 35.$$

Under pooling, the total flow rate is:

$$\min\left(\frac{\sigma_X + \sigma_Y}{\delta_X + \delta_Y}, 1\right)(\pi_X + \pi_Y) = 0.74 \times (30 + 20) \approx 36.95.$$

However, the condition $\frac{\delta_X}{\pi_X} < \frac{\delta_Y}{\pi_Y}$ does not hold, as:

$$\frac{\delta_X}{\pi_X} = \frac{160}{30} \approx 5.33, \quad \textit{and} \quad \frac{\delta_Y}{\pi_Y} = \frac{70}{20} = 3.5.$$

Despite violating the condition, pooling still increases the total treated patient flow, demonstrating that the condition is sufficient but not necessary for dominance.

Extending to Pooled Subsets

As in the pairwise case, pooling sets of pooled blood types can further contribute to alleviating shortages, as illustrated in Section 3.1. The same fundamental conditions apply: ABO-compatibility, the presence of a shortage, and a lower supply-to-demand ratio in the set of blood types receiving the reallocated supply.

Propositions 1 and 2 extend to the cases where $\mathcal{X}$ and $\mathcal{Y}$ are disjoint pooled sets rather than individual blood types. Let $\mathcal{X} \subsetneq \mathcal{B}$ and $\mathcal{Y} \subseteq \mathcal{B}\backslash X$ denote two such sets. For any $\mathcal{W} \subseteq \mathcal{B}$, define the total supply, demand, and arrival flow rates as $S_{\mathcal{W}} = \sum_{W \in \mathcal{W}} \sigma_W$, $D_{\mathcal{W}} = \sum_{W \in \mathcal{W}} \delta_W$, and $P_{\mathcal{W}} = \sum_{W \in \mathcal{W}} \pi_W$. Assume $\mathcal{X}$ and $\mathcal{Y}$ satisfy the following conditions:

1. **Compatibility:** $\mathcal{Y}$ belongs to the compatibility set of $\mathcal{X}$, i.e., $\mathcal{Y} \in \mathcal{C}(\mathcal{X}, \mathbf{Y})$.
2. **Shortage in $\mathcal{X}$:** At least one blood type in $\mathcal{X}$ has insufficient supply relative to demand, such that the pooled supply-to-demand ratio satisfies: $\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}} < 1$.

3. **Supply-to-Demand Comparison:** The supply-to-demand ratio for $\mathcal{Y}$ is greater than that for $\mathcal{X}$, i.e. $\frac{S_\mathcal{X}}{D_\mathcal{X}} < \frac{S_\mathcal{Y}}{D_\mathcal{Y}}$.

The proofs have been relegated to Appendix A for brevity, as they closely resemble the proofs presented for the case with two blood types.

Proposition 3. *When the supply-to-demand ratio of $\mathcal{Y}$ is greater than that of $\mathcal{X}$ (i.e., $\frac{S_\mathcal{X}}{D_\mathcal{X}} < \frac{S_\mathcal{Y}}{D_\mathcal{Y}}$), and both are experiencing shortages (i.e. $\frac{S_\mathcal{X}}{D_\mathcal{X}}, \frac{S_\mathcal{Y}}{D_\mathcal{Y}} < 1$), pooling is dominant if and only if the average plasma demand per patient for the blood types in $\mathcal{Y}$ is greater than that for the blood types in $\mathcal{X}$, i.e., $\frac{D_\mathcal{X}}{P_\mathcal{X}} < \frac{D_\mathcal{Y}}{P_\mathcal{Y}}$.*

Proposition 4. *When there is an excess supply in $\mathcal{Y}$ (i.e., $\frac{S_\mathcal{Y}}{D_\mathcal{Y}} > 1$) but not enough to fully eliminate the shortage in $\mathcal{X} \cup \mathcal{Y}$ (i.e. $\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}} < 1$), pooling leads to a dominant allocation if the average plasma demand per patient in $\mathcal{X}$ is less that that in $\mathcal{Y}$, i.e., $\frac{D_\mathcal{X}}{P_\mathcal{X}} < \frac{D_\mathcal{Y}}{P_\mathcal{Y}}$.*

Iterative Pairwise Pooling and Dominance Criteria

This subsection presents a proposition illustrating how the results derived in the preceding subsection can be incorporated into an iterative pairwise pooling procedure to ensure that the equity objective does not undermine pooling efficiency. The following dominance criterion is central to the this process:

Definition 4. *For two disjoint sets of pooled blood types, $\mathcal{X} \subsetneq \mathcal{B}$ and $\mathcal{Y} \subseteq \mathcal{B}\backslash\mathcal{X}$, the **dominance criteria** is satisfied if:*

$$\frac{S_\mathcal{X}}{D_\mathcal{X}} < \frac{S_\mathcal{Y}}{D_\mathcal{Y}} \quad \textit{and} \quad \frac{D_\mathcal{X}}{P_\mathcal{X}} < \frac{D_\mathcal{Y}}{P_\mathcal{Y}}.$$

Proposition 5. *In any iterative pairwise pooling procedure, if the dominance criterion is satisfied at each step where two blood types or sets of pooled blood types are combined, the procedure results in a dominant final allocation.*

Proof. In an iterative pairwise pooling procedure, changes to the supply-to-demand ratio and consequently, the total flow rate of treated patients occur only when two blood types or sets of blood types are pooled.

Step 0 (Initial Allocation)
At the initial step ($t = 0$), the total flow rate of treated patients is given by:

$$F_0 = \sum_{X \in \mathcal{B}} \frac{\sigma_X}{\delta_X} \pi_X$$

where each blood type is treated individually under ABO-identical treatment.

Step $t \geq 1$
At step t, where two sets of blood types, $\mathcal{X}$ and $\mathcal{Y}$, are pooled, the total flow rate of treated patients is given by:

$$F_t = \frac{S_{\mathcal{X}}}{D_{\mathcal{X}}} P_{\mathcal{X}} + \frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}} P_{\mathcal{Y}} + \sum_{\mathcal{Z} \in \mathcal{R}} \frac{S_{\mathcal{Z}}}{D_{\mathcal{Z}}} P_{\mathcal{Z}}$$

where, $\mathcal{R} = \mathcal{B} \backslash (\mathcal{X} \cup \mathcal{Y})$ denotes the set of all remaining blood types after removing $\mathcal{X}$ and $\mathcal{Y}$ and $\mathcal{Z}$ denotes an element of $\mathcal{R}$, which may correspond to either an individual blood type or pooled subset from prior steps.

The total flow rate after pooling $\mathcal{X}$ and $\mathcal{Y}$ will not decrease relative to the previous step ($F_t \geq F_{t-1}$) if:

$$\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}} P_{\mathcal{X}} + \frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}} P_{\mathcal{Y}} + \sum_{\mathcal{Z} \in \mathcal{R}} \frac{S_{\mathcal{Z}}}{D_{\mathcal{Z}}} P_{\mathcal{Z}} < \frac{S_{\mathcal{X}} + S_{\mathcal{Y}}}{D_{\mathcal{X}} + D_{\mathcal{Y}}} (P_{\mathcal{X}} + P_{\mathcal{Y}}) + \sum_{\mathcal{Z} \in \mathcal{R}} \frac{S_{\mathcal{Z}}}{D_{\mathcal{Z}}} P_{\mathcal{Z}}$$

Equivalently, this can be expressed as:

$$\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}} P_{\mathcal{X}} + \frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}} P_{\mathcal{Y}} < \frac{S_{\mathcal{X}} + S_{\mathcal{Y}}}{D_{\mathcal{X}} + D_{\mathcal{Y}}} (P_{\mathcal{X}} + P_{\mathcal{Y}})$$

By Propositions 3 and 4, the above inequality is true if the dominance criteria is satisfied for the sets being pooled.

Moreover, if the dominance criteria is satisfied at each step of the iterative procedure where two sets of blood types are pooled, the total flow rate of treated patients at the final step (F_T) satisfies:

$$F_T \geq F_{T-1} \geq \ldots \geq F_0$$

Thus, the pooling procedure guarantees a dominant final allocation, ensuring that the total flow rate of treated patients is never lower than in the initial ABO-identical treatment case. □

6 Simulation: Assessing the Efficiency of Iterative Pairwise Pooling Procedures

This section presents simulation results that assess the magnitude and implications of the theoretical findings. The convalescent plasma (CP) donation and demand model developed by Kominers et al. (2020) serves as a suitable framework for this analysis, as it captures how variations in average plasma demand across blood types can arise endogenously from factors such as donation

behaviors and allocation rules.[12] The primary objective here is to demonstrate how pairwise pooling procedures that fail to incorporate the dominance criterion can result in inefficient allocations, providing a representative example within a broader set of potential applications.

The CP model describes the generation of CP demand and supply within a plasma rationing system, where priority is given to clinical trial patients, followed by allocations through two plasma donation incentivization schemes. Any remaining plamsa is then assigned to a general, non-prioritized patient pool. In this simulation, samples for the non-prioritized patient pool were generated under the assumption that the clinical trial patients and incentivization scheme patients have already been served. Only samples where at least one blood type faced a shortage were included in the following analysis.

Table 3: Blood-Type Distribution by Country

	Blood Type (%)			
Country	A	B	AB	O
Brazil	39	10	3	48
India	23	34	8	35
Indonesia	23	26	7	44
Sweden	43	9	4	44
UK	38	10	3	48
USA	42	10	4	44

Blood type distributions vary significantly across countries due to genetic diversity, migration patterns and historical population dynamics, among other factors. To incorporate these variations and ensure the robustness of the simulation results, samples were generated using the blood type distributions of six countries: Brazil, India, Indonesia, Sweden, the United Kingdom, and the United States of America.[13] For each country, 10,000 samples were generated using their respective blood type distributions, as shown in Table 3.

Four different pooling procedures were then applied to pool compatible blood types in each sample. These included three variations of the iterative pairwise pooling procedure described in Section 3.2, as well as the method proposed by Kominers et. al (2021) in the most recent version of their paper. More specifically, the following pooling procedures were used:

- **Ascending:** In each iteration, the compatible blood type or subset with the lowest supply-to-demand ratio is selected for pooling, as demonstrated earlier in Section 4.2 Case A.[14]

[12] See Annex C for a detailed description of the CP plasma donation and demand model.
[13] See Annex D for a detailed description of the data simulation methodology.
[14] Formally, this corresponds to a pooling procedure where $\mathbf{R} = \{\mathcal{Y} \in \mathcal{C}(\mathcal{X}, \mathbf{Y}) | s_{\mathcal{Y}} = \min\{s_{\mathcal{Z}} \in \mathcal{C}(\mathcal{X}, \mathbf{Y})\}\}$

- **Random:** In each iteration, a compatible blood type or subset is randomly selected for pooling.[15]
- **Descending:** In each iteration, the compatible blood type or subset with the lowest supply-to-demand ratio is selected for pooling, as demonstrated earlier in Section 4.2 Case B.[16]
- **Hardest-to-Serve First:** This procedure prioritizes blood types or subsets that are hardest to serve (i.e., those with the lowest supply-to-demand ratio) and allocates as much plasma as possible to them. Unlike other methods, it evaluates all possible compatibilities between blood types, ensuring plasma is allocated to maximize coverage for the most underserved groups. For a detailed description of the procedure, see Annex E.

Table 4: Percentage of Inefficient Pooling Outcomes by Country and Procedure

	Ascending	Descending	Ramdom	Hardest-to-Serve
Brazil	7.23	7.23	7.23	7.23
India	6.90	6.90	6.90	6.90
Indonesia	6.91	6.92	6.92	6.92
Sweden	7.18	7.18	7.18	7.18
UK	7.23	7.23	7.23	7.23
USA	7.19	7.19	7.19	7.19

Frequency of Inefficient Outcomes: Table 4 summarizes the percentage of samples in which pooling procedures resulted in an inefficient outcome, reducing the total flow rate of treated patients compared to the baseline of ABO-identical treatment. Overall, 7.11% of pooling procedure outcomes were inefficient. The inefficiency rates ranged from 6.90% to 7.23%, varying by country. The Indian sample exhibited the lowest inefficiency rate, while the highest rates were observed in the Brazilian and U.S. samples.

Magnitude of Inefficiency: Table 5 summarizes the mean percentage reduction in the total flow rate of treated patients and the standard error for cases with inefficient outcomes. Across countries, mean reductions ranged from 0.60% to 1.03% fewer patients treated per unit time, with the lowest in Sweden (0.60%) and the highest in India (1.03%).[17] Additionally, as shown in Table 6, the maximum observed inefficiency resulted in up to 8.43% fewer patients treated per unit time. Maximum reductions varied across countries, from 8.43% in the UK to 5.86% in the USA, reflecting differences in pooling procedures and country-specific outcomes.

[15] Formally, this corresponds to a pooling procedure where $\mathbf{R} \sim \text{Uniform}(\mathcal{C}(\mathcal{X}, \mathbf{Y}))$

[16] Formally, this corresponds to a pooling procedure where $\mathbf{R} = \{\mathcal{Y} \in \mathcal{C}(\mathcal{X}, \mathbf{Y}) | s_{\mathcal{Y}} = \max\{s_{\mathcal{Z}} \in \mathcal{C}(\mathcal{X}, \mathbf{Y})\}\}$

[17] Given that the simulation assumes approximately 350 non-prioritized patients arrive per unit time, reductions of 0.60%, 1.03%, 5.86%, and 8.43% correspond to 2.10, 3.60, 20.50, and 29.29 fewer patients treated per unit time, respectively. See Annex D for a detailed description of the simulation setup.

Table 5: Mean Reduction in Total Flow Rate of Treated Patients by Country and Procedure (%)

	Ascending		Descending		Random		Hardest-to-Serve	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE
Brazil	0.62	0.03	0.62	0.03	0.62	0.03	0.62	0.03
India	1.03	0.05	1.03	0.05	1.03	0.05	1.03	0.05
Indonesia	0.92	0.04	0.92	0.04	0.92	0.04	0.92	0.04
Sweden	0.60	0.03	0.60	0.03	0.60	0.03	0.60	0.03
UK	0.62	0.03	0.62	0.03	0.62	0.03	0.62	0.03
USA	0.64	0.03	0.64	0.03	0.64	0.03	0.64	0.03

Impact of Incorporating the Dominance Criteria: The iterative pairwise pooling procedures, ascending, descending, and random, were re-evaluated with the inclusion of the dominance criterion. Among the previously identified inefficient outcomes, 95.55% were resolved by reverting to ABO-identical allocation under the revised procedure. However, in 4.45% of cases, the procedure instead recommended an alternative pooled configuration. For summary of dominant pooling outcomes, see Table 10 in Annex D.

Table 6: Maximum Reduction in Total Flow Rate of Treated Patients by Country and Procedure (%)

	Ascending	Descending	Ramdom	Hardest-to-Serve
Brazil	8.42	8.42	8.42	8.42
India	7.24	7.24	7.24	7.24
Indonesia	8.30	8.30	8.30	8.30
Sweden	5.98	5.98	5.98	5.98
UK	8.43	8.43	8.43	8.43
USA	5.86	5.86	5.86	5.86

Nonetheless, the results show little variation across pooling procedures because the limited number of blood types and strict ABO-compatibility constraints restrict feasible pooling configurations. As a result, different procedures often lead to the same outcome, suggesting that the choice of pooling procedure may have a limited impact in practice.

7 Relevant Literature

There exists a large body of operations research (OR) literature on blood supply chain management, focusing primarily on inventory control, wastage minimization, and the efficient use of perishable blood products. While this paper is thematically related to this work, this literature

typically frames the problem as a stochastic inventory control challenge rather than a market design or allocation problem.

Pirabán et al. (2019) provide a comprehensive survey of this literature, finding that the most frequent objective in these studies pertains to minimizing wastage and shortages of perishable blood units. For instance, Abdulwahab and Wahab (2014), Dillon et al. (2017) and Dehghani and Abbasi (2018) all incorporate ABO-compatible blood substitutions within their models, but these compatibility constraints are treated as secondary to inventory optimization. A notable exception is Duan and Liao (2014), who finds that allowing ABO-compatible substitutions along the blood supply chain increases flexibility in managing red blood cell inventory and reduces system-wide wastage due to expiration by up to 16%. However, these models are less applicable in high-demand, supply-constrained settings, such as CP allocation during a viral epidemic or pandemic, where shortages, rather than inventory management, present the primary challenge.

This paper also broadly relates to the market design literature on allocating scarce medical resources under compatibility constraints, particularly in kidney exchange, where both ABO blood type and tissue type (HLA) compatibility restrict feasible matches. Roth et al. (2004, 2005) developed and formalized kidney exchange mechanisms, establishing the foundation for compatibility-based allocation models. They demonstrated how paired exchanges (two incompatible patient-donor pairs swapping kidneys) and multi-way exchanges (larger transplant cycles) increase efficiency by expanding feasible matches when direct donation is not possible. Subsequent research has explored altruistic donor chains (e.g., Ashlagi et al., 2012; Rees et al., 2009; Roth et al., 2006), strategic behavior in exchanges (e.g., Agarwal et al., 2019; Ashlagi & Roth, 2014; Ausubel & Morrill, 2014), dynamic allocation mechanisms (Ünver, 2010), and incentive-based participation schemes (Sönmez et al., 2020), among others.

Further, several papers also examine how compatibility constraints affect the efficiency and equity in kidney allocation. For instance, Sönmez et al. (2020) propose offering priority in the deceased-donor queue, should the patient require a repeat transplant, to incentivize participation in kidney exchange. Their analysis shows that this scheme can enhance welfare and equity, particularly benefiting biologically disadvantaged blood type O patients and addressing broader inequities in access for blood type B patients. Similarly, Ashlagi et al. (2012) demonstrate that long non-simultaneous chains enhance efficiency and are especially beneficial for highly sensitized patients with limited matching opportunities due to tissue-type incompatibilities. Further, Andersson and Kratz (2020) show that allowing kidney exchange across the ABO blood barrier, enabled by the availability of immunosuppressive treatments, not only increases the total number of transplants but also reduces disparities for biologically disadvantaged blood type O patients.

While the kidney exchange literature provides a well-established framework for allocation under compatibility constraints, these insights have not yet been extended to blood supply allocation. The most closely related work is Kominers et al. (2020, 2021), who introduce a market design approach to plasma donation, developing a centralized mechanism for collecting and distributing convalescent plasma (CP) during the COVID-19 pandemic. Of particular relevance to

this paper, they propose that allowing ABO-compatible plasma donations can improve equity in blood supply distribution. To achieve this, they present an optimal pooling procedure that equalizes supply-to-demand ratios.[18] Their approach maximizes the minimum supply-to-demand ratio and minimizes disparities in service rates.

Further, as one of the first market design papers on blood allocation, this paper also relates to Han et al. (2021), who develop a general framework for blood allocation with replacement donors. Their model accommodates endogenous exchange rates, allowing patients to trade donor-provided blood for transfusions. By introducing incentive-compatible allocation mechanisms, they address inefficiencies in existing replacement donor programs and propose mechanisms that improve both equity and efficiency in blood allocation.

This paper contributes to these literatures by extending the market design approach to blood allocation, evaluating how ABO-compatible pooling procedures aimed at improving equitable access to the blood supply across blood types affect the total flow rate of treated patients.

8 Conclusion

This paper studies ABO-compatible pooling procedures in a steady-state setting, highlighting a potential trade-off between equity and total patient treatment. While pooling procedures that equalize the proportion of patients treated across compatible blood types can promote equitable access to blood resources, they may also reduce the overall number of treated patients. Simulations based on the plasma donation model of Kominers et al. (2020) indicate that, in cases where pooling leads to inefficiencies, the total number of treated patients can decrease by an average of 0.60% to 1.03%.

To address this issue, a dominance criterion for ABO-compatible pooling is derived. Specifically, when pooling two subsets of non-intersecting blood types, the final allocation will be dominant if the average plasma demand per patient is lower in the set with the lower supply-to-demand ratio (plasma-receiving blood types) than in the set with the higher ratio (plasma-donating blood types). Furthermore, this criterion can be easily integrated into iterative pairwise pooling procedures to ensure that equity considerations do not reduce the total flow rate of treated patients. As a result, it offers a simple yet practical guideline for blood banks and policymakers in designing more effective allocation mechanisms.

While these findings provide practical insights, this area of research remains in its early stages, leaving ample room for further exploration. This paper assumes a steady-state setting, whereas blood allocation in reality operates in a dynamic environment where supply and demand fluctuate—especially during viral outbreaks, when uncertainty, demand surges, and supply shocks amplify these fluctuations. Future research could examine how dynamic factors, such

[18]The authors have released multiple versions of this working paper. The pooling procedure discussed in this paper relates most closely to the pairwise iterative pooling procedure in an earlier version (Kominers et al., 2020).

as shifting demand patterns and evolving donation behaviors, impact allocative equity and efficiency. Additionally, this study focuses solely on pairwise iterative pooling; future work could explore alternative pooling procedures to better understand their implications for both efficiency and equity.

The trade-off between equity and sustaining the total flow rate of treated patients rate is a key consideration in blood allocation decisions. However, the relative priority given to each may be guided by normative considerations. The findings in this paper suggest that incorporating the dominance criterion into ABO-compatible pooling procedures can help mitigate this trade-off, ensuring that efforts to enhance equity do not come at the cost of substantial efficiency losses.

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A Proofs

Proposition 3. *When the supply-to-demand ratio of $\mathcal{Y}$ is greater than that of $\mathcal{X}$ (i.e., $\frac{S_\mathcal{X}}{D_\mathcal{X}} < \frac{S_\mathcal{Y}}{D_\mathcal{Y}}$), and both are experiencing shortages (i.e. $\frac{S_\mathcal{X}}{D_\mathcal{X}}, \frac{S_\mathcal{Y}}{D_\mathcal{Y}} < 1$), pooling is dominant if and only if the average plasma demand per patient for the blood types in $\mathcal{Y}$ is greater than that for the blood types in $\mathcal{X}$, i.e., $\frac{D_\mathcal{X}}{P_\mathcal{X}} < \frac{D_\mathcal{Y}}{P_\mathcal{Y}}$.*

Proof. Pooling is dominant if the total flow rate of treated patients exceeds that under ABO-identical treatment, or equivalently if:

$$\min\left(\frac{S_\mathcal{X}}{D_\mathcal{X}}, 1\right) P_\mathcal{X} + \min\left(\frac{S_\mathcal{Y}}{D_\mathcal{Y}}, 1\right) P_\mathcal{Y} < \min\left(\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}}, 1\right) P_{\mathcal{X}\cup\mathcal{Y}}.$$

Since both $\mathcal{X}$ and $\mathcal{Y}$ are experiencing shortages $\left(\frac{S_\mathcal{X}}{D_\mathcal{X}}, \frac{S_\mathcal{Y}}{D_\mathcal{Y}} < 1\right)$, the minimum functions simplify:

$$\frac{S_\mathcal{X}}{D_\mathcal{X}} P_\mathcal{X} + \frac{S_\mathcal{Y}}{D_\mathcal{Y}} P_\mathcal{Y} < \frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}} P_{\mathcal{X}\cup\mathcal{Y}}.$$

Expanding and rearranging the terms yields:

$$\frac{S_\mathcal{X}}{D_\mathcal{X}}\left(P_\mathcal{X} D_{\mathcal{X}\cup\mathcal{Y}} - D_\mathcal{X} P_{\mathcal{X}\cup\mathcal{Y}}\right) < \frac{S_\mathcal{Y}}{D_\mathcal{Y}}\left(D_\mathcal{Y} P_{\mathcal{X}\cup\mathcal{Y}} - P_\mathcal{Y} D_{\mathcal{X}\cup\mathcal{Y}}\right).$$

Substituting $D_{\mathcal{X}\cup\mathcal{Y}} = D_\mathcal{X} + D_\mathcal{Y}$ and $P_{\mathcal{X}\cup\mathcal{Y}} = P_\mathcal{X} + P_\mathcal{Y}$, the inequality becomes:

$$\frac{S_\mathcal{X}}{D_\mathcal{X}}\left(P_\mathcal{X}(D_\mathcal{X} + D_\mathcal{Y}) - D_\mathcal{X}(P_\mathcal{X} + P_\mathcal{Y})\right) < \frac{S_\mathcal{Y}}{D_\mathcal{Y}}\left(D_\mathcal{Y}(P_\mathcal{X} + P_\mathcal{Y}) - P_\mathcal{Y}(D_\mathcal{X} + D_\mathcal{Y})\right),$$

Simplifying and rearranging yields:

$$\left(\frac{S_\mathcal{X}}{D_\mathcal{X}} - \frac{S_\mathcal{Y}}{D_\mathcal{Y}}\right)\left(P_\mathcal{X} D_\mathcal{Y} - D_\mathcal{X} P_\mathcal{Y}\right) < 0$$

Since $\frac{S_\mathcal{X}}{D_\mathcal{X}} < \frac{S_\mathcal{Y}}{D_\mathcal{Y}}$, the inequality above holds if and only if $P_\mathcal{X} D_\mathcal{Y} > D_\mathcal{X} P_\mathcal{Y}$, or equivalently $\frac{D_\mathcal{Y}}{P_\mathcal{Y}} > \frac{D_\mathcal{X}}{P_\mathcal{X}}$. Thus, pooling is dominant if and only if the average plasma demand per patient for blood types in $\mathcal{Y}$ is greater than that for the blood types in $\mathcal{X}$. □

Proposition 4. *When there is an excess supply in $\mathcal{Y}$ (i.e., $\frac{S_\mathcal{Y}}{D_\mathcal{Y}} > 1$) but not enough to fully eliminate the shortage in $\mathcal{X} \cup \mathcal{Y}$ (i.e. $\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}} < 1$), pooling leads to a dominant allocation if the average plasma demand per patient in $\mathcal{X}$ is less that that in $\mathcal{Y}$, i.e., $\frac{D_\mathcal{X}}{P_\mathcal{X}} < \frac{D_\mathcal{Y}}{P_\mathcal{Y}}$.*

Proof. Pooling is dominant if the total flow rate of treated patients exceeds that under ABO-identical treatment, or equivalently if:

$$\min\left(\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}},1\right)P_{\mathcal{X}}+\min\left(\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}},1\right)P_{\mathcal{Y}}<\min\left(\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}},1\right)P_{\mathcal{X}\cup\mathcal{Y}}.$$

Since $\mathcal{Y}$ has an excess $\left(\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}>1\right)$ and $\mathcal{X}$ is experiencing a shortage $\left(\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}<1\right)$, the minimum functions simplify:

$$\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}P_{\mathcal{X}}+P_{\mathcal{Y}}<\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}}P_{\mathcal{X}\cup\mathcal{Y}}$$

Since $P_{\mathcal{Y}}<\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}P_{\mathcal{Y}}$ as $\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}>1$, we have:

$$\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}P_{\mathcal{X}}+P_{\mathcal{Y}}<\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}P_{\mathcal{X}}+\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}P_{\mathcal{Y}}$$

Further, by following the same steps as in Proposition 3, it can be shown that:

$$\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}P_{\mathcal{X}}+\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}P_{\mathcal{Y}}<\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}}P_{\mathcal{X}\cup\mathcal{Y}}$$

can be rearranged and simplified to yield:

$$\left(\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}-\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}\right)\left(P_{\mathcal{X}}D_{\mathcal{Y}}-D_{\mathcal{X}}P_{\mathcal{Y}}\right)<0$$

As before, this inequality holds if and only if:

$$\frac{D_{\mathcal{X}}}{P_{\mathcal{X}}}<\frac{D_{\mathcal{Y}}}{P_{\mathcal{Y}}}.$$

Thus, combining these results, if $\frac{D_{\mathcal{X}}}{P_{\mathcal{X}}}<\frac{D_{\mathcal{Y}}}{P_{\mathcal{Y}}}$, then:

$$\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}P_{\mathcal{X}}+P_{\mathcal{Y}}<\frac{S_{\mathcal{X}}}{D_{\mathcal{X}}}P_{\mathcal{X}}+\frac{S_{\mathcal{Y}}}{D_{\mathcal{Y}}}P_{\mathcal{Y}}<\frac{S_{\mathcal{X}\cup\mathcal{Y}}}{D_{\mathcal{X}\cup\mathcal{Y}}}P_{\mathcal{X}\cup\mathcal{Y}}$$

Therefore, if $\frac{D_{\mathcal{X}}}{P_{\mathcal{X}}}<\frac{D_{\mathcal{Y}}}{P_{\mathcal{Y}}}$ then pooling results in a dominant allocation. □

B Illustration: CASE B

This section presents the detailed procedural steps for CASE B from Section 4.

CASE B: Highest Supply-to-Demand Ratio Rule

$$\mathbf{R} = \{\mathcal{Y} \in \mathcal{C}(\mathcal{X}, \mathbf{Y}) \mid s_{\mathcal{Y}} = \max\{s_{\mathcal{Z}} \mid \mathcal{Z} \in \mathcal{C}(\mathcal{X}, \mathbf{Y})\}\}$$

The rule $\mathbf{R}$ selects the element $\mathcal{Y}$ from the compatibility set $\mathcal{C}(\mathcal{X}, \mathbf{Y})$ such that $s_{\mathcal{Y}}$ is the maximum supply-to-demand ratio among all elements in the compatibility set. The pooling procedure would then proceed as follows:

Step 0: $\mathbf{B^0} = \{\{A\}, \{AB\}, \{B\}, \{O\}\}$.
The supply to demand ratio are $s_O = 0.25, s_A = 0.50, s_{AB} = 0.80$ and $s_B = 1$.

Step 1: $\{O\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{O\}, \mathbf{B^0}) = \{\{A\}, \{AB\}, \{B\}, \{O\}\}$.
$\{B\} \in \mathcal{C}(\{O\}, \mathbf{B^0})\backslash\{\{O\}\}$ has the highest supply-to-demand ratio.
$\{O\}$ and $\{B\}$ are pooled together as $\{O, B\}$.
$\mathbf{B^1} = \mathbf{B^0}\backslash\{\{O\}, \{B\}\} \cup \{O, B\} = \{\{A\}, \{AB\}, \{O, B\}\}$.
The new supply to demand ratio for $\{O, B\}$ is:

$$s_{\{O,B\}} = \frac{10 + 100}{40 + 100} = \frac{110}{140} = 0.79$$

Step 2: $\{A\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{A\}, \mathbf{B^1}) = \{\{A\}, \{AB\}\}$.
$\{AB\} \in \mathcal{C}(\{A\}, \mathbf{B^1})\backslash\{\{A\}\}$ has the highest supply-to-demand ratio.
$\{A\}$ and $\{AB\}$ are pooled together as $\{A, AB\}$.
$\mathbf{B^2} = \mathbf{B^1}\backslash\{\{A\}, \{AB\}\} \cup \{A, AB\} = \{\{A, AB\}, \{O, B\}\}$.
The new supply to demand ratio for $\{A, AB\}$ is:

$$s_{\{A,AB\}} = \frac{20 + 32}{40 + 40} = \frac{52}{80} = 0.65$$

Step 3: $\{A, AB\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{A, AB\}, \mathbf{B^2}) = \{\{A, AB\}\}$ so it is not pooled with any other set.
$\mathbf{B^3} = \mathbf{B^2}\backslash\{A, AB\} = \{\{O, B\}\}$.

Step 4: $\{O, B\}$ has the lowest supply to demand ratio.
$\mathcal{C}(\{O, B\}, \mathbf{B^3}) = \{\{O, B\}\}$ so it is not pooled with any other set.

$\mathbf{B^4} = \mathbf{B^3} \backslash \{O, B\} = \emptyset$.

The procedure ends. The pooled sets are $\{A, AB\}$ and $\{O, B\}$. The corresponding supply to demand ratios are $s_{\{A,AB\}} = 0.65$ and $s_{\{O,B\}} = 0.79$

C CP Donation and Demand Model

The Kominers et al. (2020) CP model for plasma donation and demand describes a steady-state plasma rationing system that allocates some units for clinical trials, distributes the remainder through two plasma donation incentivization schemes, and channels any surplus to a non-prioritized patient pool. The two plasma donation incentivization schemes are as follows:[19]

1. **Paying it Backward:** Recovered plasma donors receive a fixed number of priority credits, redeemable in the future for top-tier treatment prioritization.

2. **Paying it Forward:** Patients without credits can secure second-tier treatment priority by pledging to donate plasma upon recovery.

The initial model assumes ABO-idential plasma donation and defines five COVID patient categories:

1. Clinical trial patients arriving at a rate of π^t

2. Credit prioritised patients arriving at a rate of π^v

3. Patients who have pledged to donate ex-ante arriving at a rate of π^f

4. Non-prioritised patients arriving at a rate of π^n

5. Patients recovering without therapy arriving at a rate of ω

The supply-to-demand ratio for plasma units allocated to clinical trails, credit-prioritized patients, pledge patients and non-prioritized patients are denoted by s^t, s^v, s^f and s^n, respectively. Assuming all treated patients recover, the flow rate of recovering patients who can potentially donate plasma is composed of the following components:

- Patients from clinical trails, with a flow rate $s^t\pi^t$,

- Credit-prioritized patients, with a flow rate of $s^v\pi^v$,

- Patients who pledged to donate ex-ante, with a flow rate $s^f\pi^f$, and

- Patients who are not part of the clinical trials, do not have credits, and have not pledged to donate, with a flow rate $s^n\pi^n + \omega$.

[19]The model is initially developed with only the paying-it-backward scheme and later extended to include the second scheme. The simulations in this paper are based on the extended model that incorporates both schemes; hence, this description focuses on the latter model.

The endogenously determined total steady-state plasma therapy supply flow rate is then given by:

$$\gamma = (p(s^t\pi^t + s^n\pi^n + \omega) + p^f s^f \pi^f)k \tag{7}$$

where, p is the probability of donation in return for credits, p^f is the probability with which a forward pledging patient donates after recovering, and k is the number of units of blood plasma that a patient can donate. It is assumed that the probability of donation (p) is the same for patients recovering from clinical trials, non-prioritized patients, and those who recover without CP therapy. However, pledging weakly increases the likelihood of donation, so $p \geq p^f$.

Each patient who donates plasma receives $v \geq 0$ priority credits, which can be redeemed for future treatment prioritization by someone of their choosing. Treatment priority is granted only to patients holding priority credits issued by plasma donors. Let v denote the prioritization credit vouchers awarded under the paying it backward scheme and v^f the prioritization credit vouchers awarded under the paying it forward scheme. The arrival flow rate of credit-prioritized patients is then given by:

$$\pi^v = p(s^t\pi^t + s^n\pi^n + \omega)qv + p^f s^f \pi^f q v^f \tag{8}$$

where q is the proportion of credits redeemed.

Given the model setup, the following proposition outlines the conditions under which all prioritised patients (i.e., clinical trial, credit-prioritized, and pledged patients) are guaranteed plasma therapy:

Proposition 4 (from Kominers et al. (2021)): Regardless of the pledged patient arrival rate π^f, so long as

$$p(k-r) \geq \frac{\pi^t}{\pi^t + \omega} \quad \text{and} \quad p^f(k - r^f) \geq 1 \tag{9}$$

it is possible to ensure that all clinical-trial, credit-prioritized patients, and pledged patients receive plasma therapy, so that

$$s^t = 1, \quad s^v = 1 \quad \text{and} \quad s^f = 1.$$

Here, $r = qv$ and $r^f = qv^f$ the credit redemption rates from the paying it backward and paying it forward schemes, respectively.

Assuming that the conditions in (9) holds, the supply-to-demand ratio for the non-prioritized

patient can then be derived as:

$$s^n = \frac{\omega p(k-r) - \pi^t(1-p(k-r)) + p^f \pi^f \left(k - r^f - \frac{1}{p^f}\right)}{\pi^n(1-p(k-r))} \tag{10}$$

This setup is then extended to account for four blood types, O, AB, A and B. While the earlier process independently models the endogenous supply and demand for each blood type, it must also consider the possibility that voucher holders may have different blood types from the original donors. To address this, the following definitions are introduced:

$$r_X = b_X \sum_{Y \in \mathcal{B}} q_Y v_Y \quad \text{and} \quad r_X^f = b_X \sum_{Y \in \mathcal{B}} q_Y v_Y^f$$

where, $\mathcal{B} = \{A, AB, B, O\}$ is the set of blood types, b_X is the probability that a patient has the blood-type X, and $q_Y v_Y$ and $q_Y v_Y^f$ are the voucher redemption rates for the backward and forward incentivization schemes for patients with blood type Y.

Further, the numerator and denominator of the supply-to-demand ratio from (10) are defined as:

$$\sigma_X := \omega_X p_X(k-r_X) - \pi_X^t(1-p_X(k-r_X)) + p_X^f \pi_X^f \left(k - r_X^f - \frac{1}{p_X^f}\right) \tag{11}$$

$$\delta_X := \pi_X^n(1-p_X(k-r_X)) \tag{12}$$

for each blood type X, where σ_X represents the steady-state net supply and δ_X represents the steady-state net demand of blood-type X plasma.

This framework is relevant to this paper as it illustrates how average plasma demand variation across bloodtypes can arise endogenously from factors such as donation behaviors and allocation rules, as reflected in:

$$\frac{\delta_X}{\pi_X^n} = (1-p_X(k-r_X)),$$

where $\frac{\delta_X}{\pi_X^n}$ represents the average plasma demand per non-prioritized patient with blood type X.

In this model, average plasma demand is influenced by the donation probabilities (p_X) and the effectiveness of the incentivization scheme ($k - r_X$). Differences in donation probabilities and the impact of allocation rules across blood types lead to heterogeneity in average demand. For instance, blood types with lower donation rates or less effective incentivization schemes will experience higher average demand.

D Simulation

D.1 Methodology and Key Assumptions

Table 7: Empirical Blood-Type Distribution by Country

	Blood Type (%)			
Country	A	B	AB	O
Brazil	39.02	10.01	3.00	47.97
India	23.01	34.00	8.01	34.99
Indonesia	23.02	25.99	7.01	43.98
Sweden	43.01	9.00	4.01	43.98
UK	38.40	10.11	3.03	48.46
USA	42.01	10.00	4.00	43.99

This section outlines the sample simulation process, including the key assumptions and methodological choice. The following assumptions were made in the data generation process:

- **Total arrival flow rate:** The total arrival flow rate ($\pi^t + \pi^v + \pi^f + \pi^n + \omega$) was fixed at 1000 patients, which could represent the weekly arrivals of patients in a region or healthcare system under consideration. This value provides a manageable and realistic scale for simulation while aligning with plausible estimates for patient inflow.

- **Blood Types:** Each patient's blood type was assigned probabilistically based on the blood type distribution specific to the respective country (See Table 3 for the country specific distribution). The resulting empirical distribution is presented in Table 7.

- **Patient Type Distribution:** Patient type were assigned probabilistically based on the following distribution: 50% of patients recover without CP (ω), 5% are voucher prioritized (π^v), 5% are forward pledging patients (π^f), 5% are assigned to the clinical trials (π^t) and the remaining 35% are non-prioritized patients (π^n). The resulting empirical distribution is presented in Table 8.

- **Probabilities of Donations:** The probability of donation in return for credits (p) was drawn independently from a uniform distribution, $p \sim U(0, 1)$. The probability with which pledged patients donated (p_f) was drawn as $p_f \sim U(p, 1)$, ensuring that $p_f \geq p$.

- **Plasma Donations:** The average number of plasma units donated by an individual (k) was drawn from a uniform distribution, $k \sim U(0, 4)$.

- **Voucher Redemption Rates:** The redemption rates for the forward and backward incentivization schemes (r, r_f) were drawn independently from a uniform distribution, $r, r_f \sim U(0, k)$.

Table 8: Empirical Patient-Type Distribution by Country (%)

	Patient Type (%)				
Country	Clinical Trial	Forward Pledge	Voucher	Non-Prioritized	Without CCP
Brazil	4.97	5.00	5.00	35.03	50.00
India	4.97	5.00	5.00	35.04	50.00
Indonesia	4.96	5.00	5.00	35.04	49.99
Sweden	4.96	5.00	5.00	35.04	50.00
UK	4.97	5.00	5.00	35.04	49.99
USA	4.96	5.00	5.00	35.04	49.99

After generating the samples, conditions for a positive spillover effect on the non-prioritized population, as specified in Proposition 4 of Kominers et al. (2021) (Equation 9), were evaluated. Among the samples satisfying these criteria, only those in which at least one blood type experienced a remaining shortage were retained. In this manner, 10,000 samples were generated for each country. Table 9 presents the total number of samples generated and the probabilities of those meeting the specified criteria, disaggregated by country.

Table 9: Total Simulated Samples and Probability for Criteria Satisfaction by Country

Country	Total Samples Generated	Probability
Brazil	450,753	0.02219
India	457,536	0.02186
Indonesia	457,333	0.02187
Sweden	452,437	0.02210
UK	450,911	0.02218
USA	453,731	0.02204

The steady-state net plasma supply (11), denoted by σ_X, and demand (12), denoted by δ_X, for each blood type $X \in \mathcal{B} = \{A, AB, B, O\}$ were then calculated as specified in the equations outlined in Annex C. For clarity, these equations are restated below:

$$\sigma_X = \omega_X p_X(k - r_X) - \pi_X^t(1 - p_X(k - r_X)) + p_X^f \pi_X^f \left(k - r_X^f - \frac{1}{p_X^f} \right) \tag{11}$$

$$\delta_X := \pi_X^n(1 - p_X(k - r_X)) \tag{12}$$

D.2 Variation in Average Demand per Patient by Blood Type

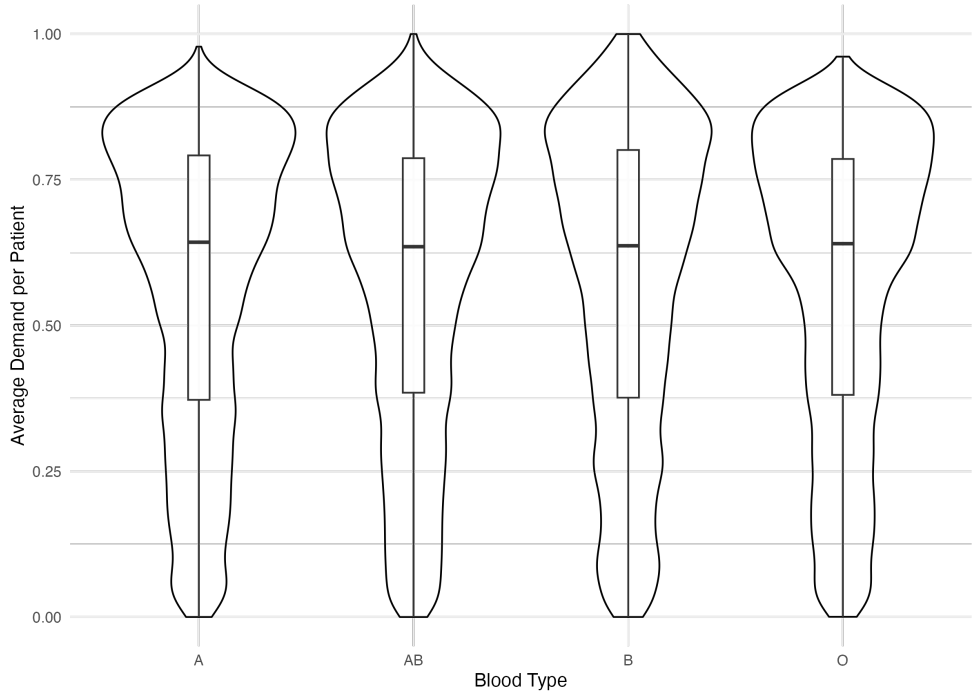
Figure 1 presents the distribution of average demand per patient across the blood types using a violin plot, overlaid with a box plot for comparison. The violin plot illustrates the density of demand values, while the box plot shows the median, interquartile range, and overall spread. As shown in the figure, there is considerable variation in average demand across blood types, driven by donation behaviors and allocation rules. Specifically, as discussed in Annex C, the average plasma demand per non-prioritized patient with blood type X is given by:

$$\frac{\delta_X}{\pi_X^n} = (1 - p_X(k - r_X)).$$

where, p_X is the donation probability in return for credits, k is the number of units that a patient can donate, and r_X is the credit redemption rate for the paying it backward scheme for blood type X.

Furthermore, under the simulation assumption that $r_X \in (0, k)$, this demand per patient is bounded between 0 and 1. Nonetheless, as evidenced by the non-dominant outcomes observed in allocations under the pooling procedures, even small differences in average demand can have significant implications for the total flow rate of treated patients.

Figure 1: Distribution of Average Demand per Patient by Blood Type



D.3 Pooling Outcomes

This section presents a summary of the pooling outcomes. Table 10 presents the distribution of dominant pooling outcomes across different pooling procedures. In 48.57% of cases, the procedures selected ABO-identical treatment, while in the remaining 51.43%, pooled allocation was dominant. The most common pooled allocations involved pooling blood types A and O (8.29%–13.59%) and blood types AB and B (9.36%).

Table 10: Distribution of Dominant Pooling Outcomes Across Pooling Procedures (%)

	Pooling Procedure (%)		
Pooled Sets	Ascending	Descending	Random
{A} {AB} {B} {O}	48.57	48.57	48.57
{A} {AB} {B,O}	5.21	5.50	9.29
{A} {AB,B} {O}	9.36	9.36	9.36
{A} {AB,B,O}	1.26	1.14	1.97
{A} {AB,O} {B}	4.04	4.08	5.19
{A,AB} {B} {O}	9.54	9.54	9.54
{A,AB} {B,O}	0.97	0.96	0.92
{A,AB,B} {O}	2.77	2.77	2.77
{A,AB,B,O}	0.98	0.89	0.92
{A,AB,O} {B}	2.18	1.99	2.05
{A,B,O} {AB}	0.65	0.01	0.30
{A,O} {AB} {B}	13.04	13.59	8.29
{A,O} {AB,B}	1.43	1.60	0.83

E Hardest-to-Serve First Pooling Procedure

This annex describes the Hardest-to-Serve First pooling procedure from Kominers et al. (2021).

Let $\mathcal{B} = \{A, B, AB, O\}$ denote the set of blood types. The supply and demand sets are defined as:

$$\mathcal{S} = \{O^S, A^S, B^S, AB^S\}$$
$$\mathcal{D} = \{O^D, A^D, B^D, AB^D\}$$

where, $\mathcal{S}$ represents the plasma supply and $\mathcal{D}$ represents the corresponding plasma demand for each blood type.

Let $\{X^D, Y^S\} \in \mathcal{S} \cup \mathcal{D}$ be an edge, meaning that plasma from blood type Y can be donated to blood-type X patients. Let $\mathbf{C}$ be the set of all such edges. Then $(\mathcal{S}, \mathcal{D}, \mathbf{C})$ is defined as the plasma compatibility graph.

For any subsets $\mathcal{D}' \subseteq \mathcal{D}$ and $\mathcal{S}' \subseteq \mathcal{S}$, define:

$$\mathcal{C}_{\mathcal{D}'}(\mathcal{S}') := \{Y^S \in \mathcal{S}' : \{Y^S, X^S\} \in \mathbf{C} \text{ for some } X^D \in \mathcal{D}'\},$$

i.e., $\mathcal{C}_{\mathcal{D}'}(\mathcal{S}')$ represents the set of blood type plasma supply in $\mathcal{S}'$ that are compatible with patients with blood-types that belong in $\mathcal{D}'$.

The supply-to-demand ratio for the blood types in $\mathcal{D}'$ when they have plasma supply from $\mathcal{S}'$ available exclusively to them is defined as:

$$s_{\mathcal{D}'}(\mathcal{S}') := \frac{\sum_{Y^S \in \mathcal{C}_{\mathcal{D}'}(\mathcal{S}')} \sigma_Y}{\sum_{X^D \in \mathcal{D}'} \delta_X}.$$

The pooling procedure then iteratively constructs partitions $S_0, \ldots, S_l$ of $\mathcal{S}$ and $D_0, \ldots, D_l$ of $\mathcal{D}$ as follows:

Step 0: Identify Blood Types Served by Infinite Supply

Define the initial demand and supply partitions:

$$D_0 := \{X^D \in \mathcal{D} : \delta_Y < 0 \text{ for some } Y^s \in \mathcal{C}_{\{\mathcal{X}^{\mathcal{D}}\}}(\mathcal{S})\},$$
$$S_0 := \{X^S \in \mathcal{S} : \delta_Y < 0 \text{ for some } Y^s \in \mathcal{C}_{\{\mathcal{X}^{\mathcal{D}}\}}(\mathcal{S})\}.$$

Here, $\delta_Y < 0$ means that there is an arbitrarily large steady-state supply of Y^S, so the patients in D_0 can be fully served. Further, the plasma of any types $X^S \in S_0$ will not be required as any type that can be served by X^S plasma can also be served by Y^S. Set the service rate for

non-prioritized patients of blood-type $X^D \in D_0$ to

$$s_X^n := 1.$$

$\vdots$

Step $l \geq 1$: Iteratively Assign Hardest-to-Serve Subsets

Given sets $S_0, S_1, \ldots, S_l$ and $D_0, D_1, \ldots, D_l$, find

$$\mathcal{D}' \subseteq \mathcal{D} \setminus \cup_{m=o}^{l-1} D_m$$

that minimizes the supply-to-demand ration:

$$s_{D'}(\mathcal{S} \setminus \cup_{m=o}^{l-1} S_m).$$

Then, define:

$$D_l := \underset{\mathcal{D}' \subseteq \mathcal{D} \setminus \cup_{m=o}^{l-1} D_m}{\arg\min} \{s_{D'}(\mathcal{S} \setminus \cup_{m=o}^{l-1} S_m)\}$$
$$S_l := \mathcal{C}_{D_l}(\mathcal{S} \setminus \cup_{m=o}^{l-1} S_m)$$

Allocate plasma from blood-types in S_l to patients with blood-types in D_l and set the service rate for the non-prioritized patients of each blood type $X^D \in D_l$ to:

$$s_X^n := \min\{1, s_{D_l}(S_l)\}.$$

If there remain unallocated demand groups, i.e., $\mathcal{D} \setminus \cup_{m=0}^{l} D_m \neq \varnothing$, then continue with Step $l+1$.